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Heavy thresholds, slepton masses and the μ term in anomaly mediated supersymmetry breaking

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Abstract: The effects of heavy mass thresholds on anomaly-mediated soft supersymmetry breaking terms are discussed. While heavy thresholds completely decouple to lowest order in the supersymmetry breaking, it is argued that they do affect the breaking terms at higher orders. The relevant contributions typically occur at lower order in the loop expansion compared to purely anomaly mediated contributions. The non decoupling contributions may be used to render models in which the only source of supersymmetry breaking is anomaly mediation viable, by generating positive contributions to the sleptons' masses squared. They can also be used to generate acceptable μ- and B-terms.

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Contents

1. Introduction

Recently, it was pointed out that in the presence of any supersymmetry-breaking hidden sector, soft supersymmetry-breaking terms are generated for the observable fields through the super-Weyl anomaly [1, 2]. Because of their origin, these supersymmetry (SUSY) breaking terms are directly related to the scaling dimension of the relevant operator.

The soft terms are most readily obtained by working in a supergravity formulation that uses the superconformal, or super-Weyl compensator, Φ, a non-dynamical chiral superfield which allows one to write a manifestly super-Weyl invariant action at the classical level [3]. Essentially the role of Φ is to compensate for the non-trivial super-Weyl transformations of different action terms, while any Φ vev breaks the super-Weyl invariance. In particular, one may choose the lowest component Φ vev to be 1, leading to a trivial modification of the action. However, for the discussion of supersymmetry breaking, we will focus on the auxiliary component of Φ. As was argued in [1], in the presence of supersymmetry breaking, this auxiliary component acquires a non-zero vev, which we denote by F, so that $\Phi \equiv 1 + F\theta^2$.

To recover the observable sector lagrangean coupled to canonical supergravity, one then rotates Φ away through a super-Weyl rescaling. For a scale-invariant observable sector lagrangean, Φ disappears altogether. However, if any explicit mass scale appears in the lagrangean, it gets rescaled by Φ as $M \to M\Phi$, so that in the presence of supersymmetry breaking, various tree-level supersymmetry splitting masses appear.

Even if there are no explicit mass scales in the classical theory, some mass scale is generated quantum mechanically. In particular, if we consider the renormalized lagrangean, the wave function renormalizations of the fields contain some cutoff dependence. The cutoff scale now appears multiplied by Φ. Expanding the wave function renormalization in F (which enters through Φ), one finds a soft supersymmetry breaking mass for the relevant field.[1]

Let us concentrate for now on the scalar masses squared. Since these masses squared enter through [1]

$$Z(\mu) = Z_0(\mu)\left(1 - \frac{1}{2}\gamma_i(\mu)\left(F\theta^2 + \text{c.c.}\right) + \frac{1}{4}\,\dot{\gamma}_i\,(\mu)\,|F|^2\,\theta^2\bar{\theta}^2\right), \qquad (1.1)$$

they have two immediately apparent features. First, the soft mass has no $\mathcal{O}(F^3)$ or higher contributions. Second, while the soft masses can be obtained through (1.1) at the UV cutoff of the theory and then evolved down to any lower scale, it is also possible to directly evaluate them at the low scale, again through (1.1). The reason is of course that the wave function renormalization already takes running effects into account. Thus, at any scale, the anomaly-mediated (AM) masses appear to be determined solely by the theory at that scale, specifically by the β-function and anomalous dimensions at that scale, with no memory of the theory at higher scales. This picture has been derived to leading order in the SUSY breaking parameter in a theory in which the only source of SUSY breaking is anomaly mediation.

If there are any additional contributions to the soft masses at high scales (such as the compactification scale), from, say, different gravitational sources, these contributions affect the running of the masses to lower scales.

Even if the only source of supersymmetry breaking in the theory is anomaly mediation, which generates soft masses near the UV cutoff of the theory which are given by (1.1), one may wonder about what happens when the theory also contains some heavy thresholds. The mass of the heavy fields, which we denote by M, is rescaled as $M \to M\Phi$. Therefore the heavy fields obtain tree-level supersymmetry splittings proportional to F. If the heavy fields interact with some of the light fields, they can then contribute to the light fields' soft masses at the loop level. Still, these contributions largely decouple [1, 4]. More specifically, the contribution of any heavy field decouples at order F^2. This should be apparent from our discussion above – since the AM soft masses can be read of the wave-function renormalizations, the effects of heavy thresholds should somehow be accounted for already. In section 2,

[1] The extraction of these soft masses is completely analogous to the method of calculating soft masses in models of gauge-mediated supersymmetry breaking through wave function renormalizations of [5, 6].

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we will show in detail how this decoupling occurs. However, we will argue that the heavy thresholds do not decouple completely. There are in fact two sources of non-decoupling.

First, the $\mathcal{O}(F^2)$ effects of the heavy threshold do not decouple if there is a light field associated with this threshold [4]. More specifically, this happens if the masses of the heavy fields are determined by some modulus, and this modulus is stabilized primarily by small supersymmetry breaking effects.

Second, if there are fields of mass M and supersymmetry splittings proportional to F that interact with the light fields, the loop-level soft masses that they induce contain contributions that are higher order in F. For example, scalar masses squared obtain contributions of order F^4/M^2. Furthermore, in many cases these contributions start at one-loop, whereas the F^2 contributions start at two-loops. Thus, if M is not much bigger than F, these contributions may be important.

Another interesting issue to address is the decoupling of D-terms. As we shall see in section 3, in anomaly mediation D-terms decouple to leading order in the SUSY breaking, unlike in the usual case. Again, however, F^4/M^2 contributions to D-terms do not decouple.

The above observations may be used to address the main phenomenological problem of the minimal anomaly-mediation scenario [1]. In this scenario, the soft breaking terms of the supersymmetric standard model (SM) are generated purely by anomaly mediation, and are therefore given by (1.1). The slepton masses squared are proportional to the SU(2), and U(1) β functions, and are therefore negative. Ref. [1] invoked additional gravitational contributions to overcome this problem. Ref. [4] proposed a solution in which the soft spectrum is modified by the presence of a light (order F) modulus, which is massless in the supersymmetric limit. Another solution involves additional Higgs doublets that generate large Yukawa couplings for the sleptons [8]. Here we will instead introduce new heavy thresholds, within one or two orders of the supersymmetry breaking scale F, and utilize the resulting F^4 soft masses to generate positive slepton masses squared. Our model, which has as its basis a simple U(1) theory, which we describe in section 3, illustrates the decoupling of order F^2 contributions and the non-decoupling of F^4 terms. It also clarifies the issue of D-term decoupling in AM scenarios, as some of the contributions we will discuss can be understood as arising from the U(1) D-term.

Alternatively, one could consider models with "matter messengers", heavy matter in vectorlike representations of the Standard Model (SM) gauge group, as in minimal gauge-mediated models [9]. However, in such models, there is no one-loop correction to the scalar superpartners mass-squared, so that the first non-decoupling contribution is order F^4/M^2 and two-loop suppressed, and thus small compared with pure anomaly-mediated masses. Adding Yukawa couplings between the messengers and SM matter fields ("matter-messenger mixing") does lead to a one-loop, order F^4/M^2 contribution, but this contribution is generically negative [10, 11]. Similarly,

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"gauge-messengers", that is, heavy fields associated with the breaking of some larger group to the SM, also give negative one-loop, order F^4/M^2 contributions to the scalar masses-squared [5]. Thus, the non-decoupling contributions we rely on are typically negative, and it is hard to construct a simple model based on them. This motivates our choice of a U(1) model, where the sign of some contributions is determined by charge assignments. Though several contributions to the masses-squared in this model will be negative, we will still be able to arrange for an overall positive contribution.

As we said above, the crucial contributions in our model are of the form F^4/M^2. In section 4, we will show how, in a modification of our model, the relevant scale M is generated dynamically from F. In fact, as we will see, it is quite easy to generate scales that are close to the SUSY breaking scale using anomaly mediation. This fact could be useful for model building purposes. In section 5, we put these pieces together and construct a modification of the SM in which sleptons obtain positive masses squared.

In section 6, we again use the same ingredients, that is non-decoupling $\mathcal{O}(F^4/M^2)$ contributions and a scale M that is generated dynamically from F, to naturally obtain a μ-term and a B-term of the correct size.

We close this Introduction with one comment. The mechanism of anomaly-mediated supersymmetry breaking exists in any theory with some supersymmetry breaking sector. In particular, it exists in any 4-dimensional theory. However, in a 4-dimensional theory, higher order, M_p-suppressed operators that couple the observable and hidden sector, typically generate tree-level soft masses for the observable fields. These are the well known "hidden sector" contributions. These contributions are larger than the anomaly mediated contributions, which are loop-suppressed. In a 4-dimensional theory, it is hard to rule out the existence of the tree-level hidden sector contributions. However, in [1], it was shown that in $d > 4$ dimensions, such direct couplings of the observable and hidden sector can plausibly be absent. Thus, while our discussion of decoupling is independent of the dimensionality of the full theory, when we move on to model building we envision a situation in which the soft terms are purely anomaly mediated, as in sequestered sector models [1]. We implicitly assume then that the full theory has more than 4 dimensions, and the appropriate cutoff scale, at which the anomaly-mediated soft masses of the observable sector are generated, is the compactification scale.

2. The effect of mass thresholds on soft masses of light fields

We would like to study the impact of supersymmetric mass thresholds on anomaly mediated SUSY breaking terms. We assume that the heavy thresholds lie above the visible sector SUSY breaking scale, F, and below the UV cutoff scale, whether it is the Planck scale or the compactification scale. The decoupling of these thresholds

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can then be addressed in terms of an effective 4-dimensional field theory. We also assume that there are no sources of SUSY breaking other than anomaly mediation. A discussion of decoupling effects at leading order in the SUSY breaking recently also appeared in ref. [4].

In the absence of heavy thresholds, slepton and squark soft masses follow directly from the wave function renormalization [1]:

$$\int d^4\theta Z\left(\frac{\mu}{\Lambda(\Phi^\dagger\Phi)^{1/2}}\right) Q^\dagger Q = \\ = \int d^4\theta Z\left(\frac{\mu}{\Lambda}\right)\left(1 - \frac{1}{2}\gamma(F\theta^2 + \text{c.c.}) + \frac{1}{4}(\partial_t\gamma)\left|F\right|^2\theta^2\bar{\theta}^2\right) Q^\dagger Q\,. \tag{2.1}$$

Here $\Phi = 1 + F\theta^2$ is the super-Weyl compensator, Λ is the UV cutoff, $t = \ln\mu$, and the anomalous dimension $\gamma = \partial \ln Z/\partial \ln\mu$. Rescaling the fields

$$Q \to Z^{-1/2}\exp\left(\frac{1}{2}\gamma F\theta^2\right) Q\,, \tag{2.2}$$

to obtain canonical kinetic terms, and expanding the remaining exponent yields the following soft mass for the light field Q:

$$m_s^2 = -\frac{1}{4}\left|F\right|^2(\partial_t\gamma)\,. \tag{2.3}$$

We see that the soft masses are determined by the anomalous dimensions of the light fields. Thus, the soft masses (as well as other soft parameters) are largely insensitive to the details of the high energy theory.

However, in the presence of heavy mass thresholds, there may be additional contributions to the soft masses of the light fields. The theory contains explicit mass parameters which determine the heavy threshold. Due to the scaling anomaly, all such parameters will be accompanied by the compensator field Φ. The heavy fields then acquire tree-level supersymmetry-breaking mass splittings. As these fields are integrated out, their supersymmetry-breaking splittings may in principle generate soft terms for the light fields, through gauge or Yukawa interactions. We would like to understand to what extent such thresholds affect the masses of the light fields, and in particular, whether their effects completely decouple.

To study these questions, it will prove convenient to distinguish between two possibilities. One is that the heavy threshold, or in other words, the mass of the heavy fields associated with this threshold, appears as an explicit mass term in the lagrangean. The second is that the mass of the heavy field is given by the expectation value of some dynamical field. These two cases are different because vevs are determined dynamically and may depend on non-supersymmetric effects.

Let us first consider the case that the masses of all heavy fields arise from explicit mass terms in the superpotential. As explained above, any mass term M should be promoted to a superfield-valued $X = M\Phi$. We can then obtain the soft masses of the light fields at a scale $\mu < M$ from the relevant wave function renormalization $Z(\mu)$. Assume for simplicity that Z depends on a single coupling. Then, from the 1-loop relation

$$\partial_t Z = \frac{\alpha}{\pi} c \,, \tag{2.4}$$

where c is the appropriate quadratic Casimir, we first solve (2.4) for Z in the low energy theory, and then match boundary conditions at M. Using arguments of holomorphy and R-symmetry [5, 6], we then make the substitutions $M^2 \to X^\dagger X$ and $\Lambda^2 \to \Lambda^\dagger \Lambda$. Note that both X and Λ are superfield valued and contain the compensator Φ. This yields

$$Z\left(\alpha(\mu, X), \alpha(X)\right), \tag{2.5}$$

where,

$$\alpha(X)^{-1} = \alpha(\Lambda)^{-1} + \frac{b_H}{4\pi} \ln \frac{X^\dagger X}{\Lambda^\dagger \Lambda} = \alpha(\Lambda)^{-1} + \frac{b_H}{4\pi} \ln \frac{M^2}{\Lambda^2}\,, \tag{2.6}$$

$$\begin{aligned} \alpha(\mu, X)^{-1} &= \alpha(X)^{-1} + \frac{b_L}{4\pi} \ln \frac{\mu^2}{X^\dagger X} \\ &= \alpha(\Lambda)^{-1} + \frac{b_L}{4\pi} \ln \frac{\mu^2}{\Lambda^2 \Phi^\dagger \Phi} + \frac{b_H - b_L}{4\pi} \ln \frac{M^2}{\Lambda^2}\,, \end{aligned} \tag{2.7}$$

where b_H (b_L) is the one-loop beta function above (below) the scale M. Note that the Φ dependence cancels between X and Λ in the last term of (2.7). Therefore, the only contribution to the soft-mass squared, which is $\mathcal{O}(F^2)$, comes from the second term in the second line of eq. (2.7). Since this contribution is not related to the threshold M, the low-energy theory is insensitive to the mass splittings associated with the heavy threshold.

It is important to stress that while the previous discussion can be generalized to all orders in the coupling constants, it only holds to leading order in supersymmetry breaking, that is, $\mathcal{O}(F^2)$ corrections to the scalar-masses squared. We will return to the question of higher order corrections in the end of this section.

As we just saw, the crucial point for the decoupling of the heavy fields is that their mass scale M is rescaled by Φ in the same way that Λ is, so that the Φ dependence cancels in the ratio. This implies a specific relation between the SUSY masses and the soft masses of the heavy fields. We will refer to masses that are obtained by the rescaling of some mass-scale by Φ as being "aligned". For example, the mass of a chiral field is aligned if

$$\frac{\left| m^2_{scalar} - m^2_{fermion} \right|}{m_{fermion}} = F\,, \tag{2.8}$$

and $\mathrm{Str} m^2 = 0$ in a supermultiplet.

The decoupling of the heavy threshold which we just saw may be understood as a cancellation between the anomaly-mediated contribution and the contribution of the heavy fields. To see that explicitly, consider an example in which the heavy fields are N_f pairs of fundamentals and antifundamentals of an SU(N) gauge group, with supersymmetric mass M. We also assume that the light fields transform as fundamentals of the same SU(N). Consider then the soft masses of the light fields, evaluated at a scale $\mu > M$,

$$m^2(\mu)_{\mathrm{anomaly-mediated}} = 2c_0 b_H \frac{\alpha^2}{16\pi^2} F^2 \,, \tag{2.9}$$

with b_H and c_0 the one-loop beta function and one-loop anomalous dimension coefficients, respectively. To evaluate these masses at low-scales, we should evolve them down to the low-scale, but the only thing that evolves in (2.9) is the coupling α. However, as we go below the threshold M, we also need to integrate out the heavy fields, and they contribute to the soft masses of the light fields. Their contribution is the usual gauge-mediated contribution,

$$m^2(\mu)_{\mathrm{gauge-mediated}} = 2c_0 N_F \frac{\alpha^2}{16\pi^2} F^2 \,, \tag{2.10}$$

Adding the contributions (2.9) and (2.10), we find, at $\mu < M$,

$$m^2(\mu) = 2c_0\, b_L \frac{\alpha^2}{16\pi^2} F^2 \,, \tag{2.11}$$

since $b_L = b_H + N_f$. This is precisely the soft mass we would obtain when calculating directly in the low-energy theory below the threshold M, ignoring the heavy fields altogether. We see that the contribution of the heavy fields through anomaly mediation, which is proportional to N_f and arises through their contribution to the beta function, exactly cancels their contribution through gauge mediation. Our discussion can be easily generalized to models with Yukawa couplings between light and heavy fields.

This cancellation may also be seen diagrammatically in the Pauli-Villars regularization scheme. The regulator fields always have aligned masses $\Lambda\Phi$, because an explicit mass term must be put in for them. Now, when we consider any diagram contributing to the soft masses, with heavy fields running in the loop, there will be an analogous diagram with the regulator fields running in the loop. We can assume that the real fields have mass M, the regulators have mass Λ, and treat the splittings MF and ΛF as insertions. Then, to order F^2 (for which there is no dependence on the heavy mass), the heavy fields and the regulator fields have exactly opposite contributions to the soft masses. The contributions are guaranteed to be of exactly the same magnitude because the masses of both the heavy fields and the regulators are aligned.

Now let us consider the possibility that some SUSY masses originate from non-zero vevs. Take X to be a field which acquires a vev in the supersymmetric limit. Fields that couple to X obtain a SUSY preserving mass $C\langle X\rangle$, and soft mass splittings CF_X, with C some constant that depends on different couplings.[2] Such heavy fields decouple from the effective theory of the light degrees of freedom if their masses are aligned, that is, if $F_X = XF$. In the limit of unbroken SUSY, we write $X = v+\delta x$, where v is a background superfield whose lowest component is just the vev of X, while δx is the fluctuating part of X whose vev vanishes in the supersymmetric limit. Since v is determined by a combination of mass parameters in the superpotential, upon supersymmetry breaking, it acquires an F-term which is automatically aligned with the scalar component, $v \to v(1 + F\theta^2)$. On the other hand, when supersymmetry is broken, δx can obtain both a scalar vev and an auxiliary component vev, that are determined by non-holomorphic effects and thus are not necessarily aligned. Thus we need to understand under which conditions the vevs can become significantly non-aligned. The lagrangean for δx has the form

$$\int d^4\theta \ \left[(1 - m_x^2\theta^4)(\delta x)^\dagger(\delta x) + \Big(v(\Phi^\dagger - m_x^2\theta^4)(\delta x) + \text{c.c.}\Big)\right] + \\ + \left(\int d^2\theta \frac{1}{2} m\,(\Phi - a\theta^2)(\delta x)^2 + \text{c.c.}\right), \tag{2.12}$$

where m is the mass of δx in the supersymmetric limit, while a and m_x^2 are soft mass parameters generated by anomaly mediation. Note that while we can treat m as a free parameter (in a given model, it is determined by the detailed form of $W(X)$), a and m_x are suppressed relative to F by a one loop factor, $a \sim m_x \ll F$. We see that the radiative soft parameters in the lagrangean will modify X and F_X. Solving for $F_{\delta x}$ and extremizing the potential we find for the corrected values of X and F_X at the extremum

$$X = v\frac{m^2 + ma}{m^2 + mF + m_x^2 + ma}\,, \qquad F_X = v\frac{m^2 F + mm_x^2}{m^2 + mF + m_x^2 + ma}\,. \tag{2.13}$$

It is clear that in the limit $m \to \infty$ we recover the results obtained for explicit mass parameters in the lagrangean as should have been expected. On the other hand, in the limit $m \to 0$ the extremum is found at $X = F_X = 0$ independently of the value of m_x^2. This point may in fact be the maximum of the potential if $m_x^2 < 0$. In such a case there may exist a local minimum at non-zero X if m_x^2 changes sign as a function of v [4]. Moreover, a local minimum may exist at non-zero X for any sign of m_x^2 if cubic and higher order superpotential terms for δx are taken into account [4]. In any case, as we show and quantify momentarily, for small m alignment is lost in a vacuum with non-vanishing X.

[2]For simplicity we assume here that the coupling is renormalizable, but it is easy to repeat the following discussion for the general case.

Assuming that the eigenvalues of the X mass matrix are positive, so that (2.13) gives a minimum of the potential, we can easily check whether the corrected values of X and F_X are aligned,

$$\frac{F_X}{X} = F\left(1 - \frac{a}{m} + \frac{a^2}{m^2} + \frac{m_x^2}{mF} + \cdots\right), \tag{2.14}$$

where the dots stand for higher order terms in m_x^2 and a. We see that alignment is approximately preserved if

$$a \sim \frac{y^2}{16\pi^2} F \ll m\,, \tag{2.15}$$

where y stands for the couplings of X. Note that for small m, m_x^2 gives a subleading contribution since it appears at two loops whereas a appears at one loop. Moreover, if X interacts sufficiently weakly (so that a, which is one loop suppressed, is sufficiently small), it is possible that the mass of δx is significantly below the SUSY breaking scale, $m \ll F$, yet alignment holds to a good approximation. We therefore conclude that heavy fields whose masses are generated by the vevs of dynamical fields decouple so long as all mass parameters (including the masses of the fields which obtain vevs) satisfy the hierarchy (2.15).

As we mentioned above, our discussion so far only involved the scalar masses-squared to leading order in the SUSY breaking, $\mathcal{O}(F^2)$. We do not expect anomaly-mediation to generate contributions that are higher order in F. To have the correct dimension, such corrections should be of the form $(F/\Lambda)^n F^2$, where n is even and Λ is a physical cutoff associated with the compactification scale.[3]

In contrast, when heavy fields of mass M are present in the theory, with supersymmetry breaking mass splittings, they generally generate contributions to the soft masses of the light fields that are of order F^4/M^2. In the presence of Yukawa or gauge interactions between heavy and light fields, such contributions, which we call "light-heavy mixing", may even appear at one loop. Thus, the decoupling of heavy thresholds is not complete even when the masses of the heavy fields are aligned. The decoupling holds to leading order in F^2, where there is complete cancellation between anomaly-mediated contributions and light-heavy mixing but at order F^4/M^2 and higher, there is no analogous cancellation simply because there is no anomaly-mediated contribution.

This non-decoupling can again be seen by considering the regulator fields as above. We can calculate the contributions of both the dynamical fields, whose mass is M, and the regulator fields, whose mass is some physical UV cutoff Λ by treating the SUSY breaking parameter F as a small insertion. However, subleading contributions

[3] As always in our discussions in this paper our statement is limited to the case that there are no significant bulk gravity contributions to the soft masses. These necessarily contribute to the masses at the compactification scale and thus alter the UV boundary conditions on the masses. As a result the usual formulae for anomaly mediated soft masses cannot be applied. In other words, the UV physics does not decouple.

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in F are suppressed by some power of F/M in the case of the dynamical fields, and by the same power of F/Λ in the case of the regulators. For very large Λ, the regulator contributions are negligible, and can no longer cancel the contributions of the dynamical fields. Indeed, the F^4/M^2 contributions will play an important role in the models we will discuss in the following sections. We will use these contributions to generate positive masses-squared for the standard-model sleptons.

3. A U(1) model

We will now construct a simple U(1) theory which illustrates the discussion of the previous section. The U(1) is higgsed, with some fields becoming massive. These massive fields then generate one-loop $\mathcal{O}(F^4/M^2)$ contributions to the soft masses of the remaining light fields through gauge and Yukawa interactions. These contributions do not decouple. Some of the relevant contributions can be viewed as arising from the D-term. Different U(1)-charged heavy fields receive different anomaly-mediated contributions to their soft masses, and as a result a non-zero U(1) D-term is generated. Generally, such D-terms do not decouple from the low energy theory even when the heavy fields are integrated out [7]. As we will see in the case of anomaly mediation, the D-term decouples to leading order in the SUSY breaking F^2, but not at higher orders.

Our U(1) theory consists of the fields $h_\pm$, $\chi_\pm$, and $l_\pm$, with U(1) charges ± 1, as well as the gauge singlets S, $n_{i=1,2}$, with the superpotential,

$$W = S\,(\lambda_1\, h_+ h_- - M^2) + y_1\, n_1\, h_+\, \chi_- + y_2\, n_2\, h_-\, \chi_+\,, \tag{3.1}$$

where λ_1, $y_{i=1,2}$ are couplings. The first term is needed to break the U(1). As we will see the last two terms with different Yukawa couplings, y_1 and y_2, are required to generate positive soft masses squared for some light scalars. With this superpotential all fields except $l_\pm$ acquire mass, either through the Higgs mechanism or through Yukawa interactions. We will eventually identify $l_\pm$ with standard model fields.

We assume that the only source of supersymmetry breaking in the theory is gravitational, through anomaly-mediation. We can then can keep track of supersymmetry-breaking effects by rescaling $M \to M\Phi$, with $\Phi \equiv 1 + F\theta^2$ as before. We also assume that $F \ll M$. The potential is then given by:

$$V = \left|\lambda_1\, h_+ h_- - M^2\right|^2 + \lambda_1^2\, |S|^2\, \left(|h_+|^2 + |h_-|^2\right) - 2M^2\, F\,(S + S^*) + \cdots\,, \tag{3.2}$$

where we left out terms that involve $n_{1,2}$, $\chi_\pm$, as we will be interested in a (potentially local) minimum where these fields do not obtain vevs. Note that because supersymmetry is broken, there is a tadpole for the scalar component of S, so that it develops a vev proportional to F,

$$S = -\frac{1}{\lambda_1} F + \mathcal{O}\left(\frac{F^3}{M^2}\right)\,, \tag{3.3}$$

whereas $h_\pm$ obtain vevs $h = M/\sqrt{\lambda_1} + \mathcal{O}(F^2/M)$. The U(1) is then Higgsed by the $h_\pm$ vevs, and at tree level we obtain a heavy vector multiplet with a vector of mass $M_{SUSY} = 2eh$, a scalar of mass $m^2 = M_{SUSY}^2 + 2S^2$, and two fermions of masses $\sqrt{M_{SUSY}^2 + S^2/4} \pm S/2$, where e is the U(1) gauge coupling. In addition, χ_- mixes with n_1 to give a chiral multiplet with a fermion of mass $y_1 h$ and scalars of masses-squared $y^2h^2 \pm y\lambda_1 Sh$, and similarly for the pair χ_+, n_2.

At low energies the theory contains only the fields $l_\pm$, with no renormalizable interactions. However, soft masses are generated for the $l_\pm$ scalars through gauge and Yukawa loops containing the heavy fields. As discussed in the previous section, to leading order in supersymmetry breaking, namely, order F^2, these soft masses vanish. We can see that by working directly in the low energy theory, which contains no renormalizable interactions, so that the anomalous dimensions of $l_\pm$ vanish. We will return to this point shortly and see how this can be understood from the point of view of the full theory.

However, even at one loop, both the heavy gauge multiplet and the heavy chiral fields generate contributions to the $l_\pm$ soft masses starting at order F^4/M^2.[4] Specifically, the gauge multiplet one-loop contribution to the $l_\pm$ scalar mass-squared is given by

$$m^2_{\rm gauge} = q^2 \frac{1}{16\pi^2} 4e^4 h^2 \left[\ln\left(1 + \frac{x^2}{2e^2}\right) - \frac{8x}{\sqrt{16e^2 + x^2}} \ln\left(\sqrt{1 + \frac{x^2}{16e^2}} + \frac{x}{4e}\right)\right], \tag{3.4}$$

where $x \equiv S/h$, and q is the relevant U(1) charge, which for $l_\pm$ is $q = \pm 1$. This contribution arises purely from the U(1) gauge interactions,

The contribution of the heavy matter fields $\chi_- - n_1$ to the $l_\pm$ scalar mass-squared is:

$$\begin{aligned} m^2_y = & -\frac{q}{64\pi^2} y^4 h^2 \left(1 + \frac{x^2}{2e^2}\right)^{-1} \times \\ & \times \left[\left(2 + \frac{x}{y} + \frac{x^3}{y^3}\right) \ln\left(1 + \frac{x}{y}\right) + \left(2 - \frac{x}{y} - \frac{x^3}{y^3}\right) \ln\left(1 - \frac{x}{y}\right)\right], \end{aligned} \tag{3.5}$$

with $y = y_1$, and where again, for $l_\pm$, $q = \pm 1$. This contribution, although we denote it by the subscript y for the Yukawa superpotential coupling, arises from both superpotential interactions and D-term interactions. Similarly, the contribution of the heavy matter fields $\chi_+ - n_2$ is

$$m^2_{y_2} = -\, m^2_y|_{y=y_2} \,. \tag{3.6}$$

Note the relative minus sign between (3.5) and (3.6), which arises from the opposite signs of χ_+ and χ_-.

[4] Recall that unlike F^2 terms, such contributions to the soft masses cannot be read off the wave-function renormalizations.

The contribution of (3.5) and (3.6) can also be understood as arising from the U(1) D-term. The fields $h_\pm$ obtain soft-masses from one-loop diagrams with $\chi_\pm - n_{1,2}$ running in the loop. Since h_+ and h_- have different superpotential couplings ($y_1 \neq y_2$), their soft masses are also different. As a result, the h_+ and h_- vevs are shifted by different amounts, so that the U(1) D-term is non-zero and proportional to the difference between the h_+ and h_- soft masses squared. This non-zero D-term then leads to soft masses for $l_\pm$, proportional to their U(1) charges, which are precisely given by (3.5).

Let us now return to the F^2 contributions to the soft masses. As we already mentioned, if we work directly in the low-energy theory, which contains only $l_\pm$ with no interactions, the order-F^2 anomaly-mediated soft masses, can be read off the (trivial) wave function renormalizations of $l_\pm$ and therefore vanish. We could however try to derive this result starting from the full theory, above the scale of the heavy fields.

In the UV theory, the wave function renormalizations of $l_\pm$ depend on the gauge coupling, and lead to non-zero, gauge-coupling dependent soft masses for $l_\pm$, which we denote by m_H^2. Once we go below the heavy threshold, we have to add to m_H^2 the contribution of the gauge multiplet to the soft masses which comes from loops involving the heavy gauge multiplet m_G^2, and m_G^2 is the analogue of (3.4). However, it only comes in at the two-loop level, since at one loop there can be no F^2 contribution to the soft masses squared [5]. As was shown in [5], m_G^2 can also be read off the supersymmetric wave function renormalizations. Adding these two contributions, m_G^2 precisely cancels m_H^2.

The Yukawa-dependent contribution to the $l_\pm$ soft masses is a bit trickier, since the $l_\pm$ wave function renormalizations do not depend on the Yukawa couplings. As discussed above, the Yukawa dependence enters through the D-term. As long as h_+ and h_- have different soft masses, the D-term is non-zero, and leads to soft masses for $l_\pm$. The relevant quantities to consider are then the $h_\pm$ soft masses. Again, we can read these off the $h_\pm$ wave function renormalizations in the full theory, but as we integrate out the heavy fields $\chi_+ - n_2$ and $\chi_- - n_1$, we obtain $\chi_\pm - n_{1,2}$ loops which precisely cancel the original contribution. Thus, the D-term decouples to order-F^2, since the radiative contributions to the $h_\pm$ soft masses vanish in the low-energy theory. In fact, it would be surprising if D-terms generated F^2 contributions to soft masses through anomaly-mediation. We expect to be able to obtain anomaly-mediated F^2 soft masses from wave function renormalizations, which certainly cannot capture D-term contributions. It is therefore reassuring to find that the D-term decouples at order F^2. This is in contrast to the usual case [7] where the leading order SUSY breaking effects do not decouple. As we have seen, however, there are non-decoupling effects at order $\mathcal{O}(F^4/M^2)$.

We would eventually like to identify l_+ with the standard model leptons, and to use the contributions we found to its soft mass, m^2_{gauge} and m^2_y, to compensate for the negative masses squared which the sleptons obtain in the minimal anomaly-mediated scenario [1]. However, m^2_{gauge} and m^2_y are proportional to F^4/M^2, so that would involve tuning the ratio of two unrelated scales, F and M. In the next section we will present a model in which a "large" scale $M \sim F/\lambda_0$, where λ_0 is some Yukawa coupling, is generated dynamically.

4. A two step model

Our goal is to find a mechanism which would naturally generate a mass scale which is somewhat larger than the SUSY breaking scale, yet parametrically is of the same order. In fact the model considered in the previous section suggests a way to achieve this goal. Our main observation is that the field S obtained a small vev of the order F/λ_0. For relatively small λ_0 this scale may be sufficiently large, so that the S threshold is supersymmetric, yet it is not parametrically large, and can therefore play the role of the mass scale M of the previous section. Thus we are lead to the modification of our model which has the following superpotential[5]

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\,(\lambda_1 h_+ h_- - \lambda_2 X^2) + y_1 n_1\, h_+\, \chi_- + y_2 n_2\, h_-\, \chi_+\,. \qquad (4.1)$$

We now describe the role of the various terms in this superpotential. The last three terms are exactly the same as in the superpotential (3.1) with the substitution $M^2 \to \lambda_2 X^2$. This substitution is justified since X is heavy. Moreover, all other fields in the above superpotential become heavy at the minimum at least in the sense of eq. (2.15). If X obtains a non-vanishing vev, this sector of the theory will generate soft masses for $l_\pm$ exactly as in the previous section.

The first term in the superpotential (4.1) is designed to produce an X vev of the order F/λ_0. The large scale $\tilde{M}$ is necessary to generate this vev as well as other field vevs and eventually leads to the breaking of the U(1) gauge symmetry. Such a large mass parameter may be naturally generated by various strong coupling effects in the microscopic theory. It is important, however, that our results are insensitive to the precise value of $\tilde{M}$.

Let us now analyze the model. Upon supersymmetry breaking the scalar potential has the form (neglecting the couplings to $\chi_\pm$)

$$\begin{aligned} V = &\left|\lambda_1 h_+ h_- - \lambda_2 X^2\right|^2 + |\lambda_1 S h_+|^2 + |\lambda_1 S h_-|^2 + \\ &+ \left|-2\lambda_2 S X + \lambda_0 n^2 - \tilde{M}^2\right|^2 + |2\lambda_0 X n|^2 - (2\tilde{M}^2 F X + \text{c.c.})\,. \end{aligned} \qquad (4.2)$$

We wish to separate our analysis in two stages. First we set $\lambda_2 = 0$ and find vevs for n and X using our results from the previous section. Since these fields are heavy at the minimum we then assume that their vevs are not significantly shifted in the full theory. We will discuss under which conditions this assumption is true

[5] This superpotential is the most general one preserving a $\mathrm{U}(1)_R$ symmetry.

shortly. Then we turn on λ_2 and integrate out the heavy fields X, and n, and then we consider an effective theory for the lighter fields S, h_+ and h_-. Using our results from the previous section we find that $X = F/\lambda_0$. It is also easy to find that $F_X = F^2/\lambda_0$. The superpotential for the lighter fields now acquires the form (3.1) with the substitution $M = \sqrt{\lambda_2}X$. Note that the value of mass scale $\tilde{M}$ is indeed irrelevant for the dynamics of the lighter degrees of freedom, S and $h_\pm$. We now have to check that the mass parameter M can be promoted to a superspace valued background superfield with an F-term expectation value MF. Indeed,

$$\sqrt{\lambda_2}X = \sqrt{\lambda_2}\frac{F}{\lambda_0} + \sqrt{\lambda_2}\frac{F^2}{\lambda_0}\theta^2 = \frac{\sqrt{\lambda_2}F}{\lambda_0}(1 + F\theta^2)\,. \tag{4.3}$$

Thus we can use our results from the previous section for the soft masses of $l_\pm$ by substituting $M \to \sqrt{\lambda_2}F/\lambda_0$.

Finally, let us turn to the conditions for the existence of the desired minimum in the model (4.1). We note that the model possesses a flat direction parameterized by the S vev. In the limit of unbroken SUSY and in the region of the moduli space of interest S is heavy and can not acquire a vev until $h_+ \sim h_- \to 0$. However, when we turn SUSY breaking on, it is possible that one mass eigenvalue for the two real fields in the S supermultiplet will become negative, and the local minimum will not exist. We should, therefore, require that the F-type SUSY violating masses for S are much smaller that the supersymmetric contribution to the mass. For the supersymmetric mass we have $m_S^2 = 2\lambda_0^{-2}\lambda_1\lambda_2F^2$, while the soft mass is $m_F^2 = \sqrt{2}\lambda_1 F_h = \sqrt{2\lambda_0^{-2}\lambda_1\lambda 2F^2}$. We easily see that a local minimum exists if

$$\frac{\lambda_0}{\sqrt{\lambda_1\lambda_2}} \ll 1\,. \tag{4.4}$$

Note that the combination of couplings in (4.4) is exactly the quantity x which enters formulae (3.4) and (3.5) for the soft masses. We performed a numerical minimization of the scalar potential, and verified that the local minimum exists for a range of parameters when the ratio in (4.4) is of order or smaller than 0.1.

5. Correcting the slepton masses

As was pointed out in [1], the minimal AM scenario in which the sole origin of the superpartner masses is anomaly mediation gives negative slepton masses squared. Thus extra contributions to the slepton masses squared are required. We will now use the model we constructed in the last two sections to generate positive contributions to these masses. To do that we augment the SM gauge group by the U(1) of the model we described earlier, and charge the lepton fields under this U(1), with U(1) charge +1. Our starting point is then the theory described in section 3, with six copies of the field l_+ corresponding to the six SM lepton fields: $l^i_+ = (1, 2, -1/2, +1)$,

$l_+^{i+3} = (1,1,-1,+1)$, where $i = 1\ldots3$ is a generation index, and the parenthesis indicate the SM$\times$U(1) representation. Similarly, we also take six copies of the fields χ_-, with $\chi_-^i = (1,2,1/2,-1)$, and $\chi_-^{i+3} = (1,1,1,-1)$, and six copies of the field n_1 with $n_1^i = (1,2,-1/2,0)$, and $n_2^{i+3} = (1,1,1,0)$. It is easy to check that with this field content there are no SM$\times$U(1) anomalies. The field content of the model is summarized in table 1.

The superpotential is then given as in (4.1),

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\,(\lambda_1 h_+ h_- - \lambda_2 X^2) + \\ +y_1 \sum_{i=1}^{6} n_1^i\, h_+\, \chi_-^i\,, \tag{5.1}$$

except that so far we have not added the fields l_-, χ_+ and n_2, so that the superpotential term containing them does not appear. The fields χ_-^i, n_1^i become heavy, with mass $y_1 h$. As discussed in section 2, the sleptons then obtain the following soft masses

$$m^2_{slepton} = m^2_{gauge} + 9m^2_y|_{y=y_1}\,, \tag{5.2}$$

where m^2_{gauge} and m^2_y are given in (3.4) and (3.5) with $q = 1$.

	SM	U(1)
S	$(1,1,0)$	0
X	$(1,1,0)$	0
n	$(1,1,0)$	0
h_+	$(1,1,0)$	1
h_-	$(1,1,0)$	-1
$L \equiv l_+^{i=1..3}$	$(1,2,-1/2)$	1
$\chi_-^{i=1..3}$	$(1,2,1/2)$	-1
$n_1^{i=1..3}$	$(1,2,1/2)$	0
$\bar{l} \equiv l_+^{i=4..6}$	$(1,1,-1)$	1
$\chi_-^{i=4..6}$	$(1,1,1)$	-1
$n_1^{i=4..6}$	$(1,1,-1)$	0

Table 1: The field content of the model do not show SM fields that are neutral under the U(1)).

Let us now discuss the different mass contributions. Recall that the heavy matter contribution to the l_+ mass, m^2_y, is proportional to the charge of l_+, whereas m^2_{gauge} is proportional to the square of the charge. In addition, m^2_{gauge} is always negative. Thus, if we want to generate positive masses squared for all sleptons, they all have to have U(1) charges of the same sign. Thus, we choose all leptons to have U(1) charge $+1$, and identify them with the fields l_+. To cancel anomalies, we then add fields of the type χ_-, of U(1) charge -1, and n_1. These fields then become heavy, and at one loop generate the contribution $m^2_y|_{y=y_1}$. Unfortunately, however, this contribution is also negative! Examining eq. (3.6), we see that in order to get a positive contribution, we need heavy fields of opposite U(1) charge running in the loop, that is, fields of the type χ_+ (and their n_2 partners). Again anomaly considerations then require the presence of additional fields l_-. Unlike the χ's and the n's, these fields remain light, so that the simplest possibility is to identify them with some of the SM fields. We are therefore led to charging additional SM fields under the U(1), with charges that are opposite in sign to the lepton charges. We will now discuss two possibilities of doing so. One in which the down antiquarks have charge -1, and the other in which the first and second generation quarks have charge -1. While we will be able to generate positive masses squared for the sleptons, we see from the required matter content that probably the most disappointing aspect of our model is that it can not be made consistent with grand unification. As explained above, the lepton fields all have the same U(1) charge, and some other SM fields should have the opposite U(1) charge. This automatically excludes both SU(5) and SO(10) unification. Moreover, the additional matter fields do not come in GUT representations, and even gauge coupling unification requires the introduction of extra matter at intermediate scales.

5.1 U(1) charged leptons and down antiquarks

As discussed above, in order to obtain positive slepton masses-squared, we need heavy fields of positive U(1) charge running in the loop. We can achieve that by identifying the SM down antiquarks with the fields l_-. Thus, there are three copies of the field l_-: $l^i_- = (\bar{3}, 1, -1/3, -1)$, which are accompanied by three copies of the field χ_+ with $\chi^i_+ = (3, 1, 1/3, 1)$ and three copies of n_2 with $n^i_2 = (\bar{3}, 1, -1/3, 0)$. This additional field content is summarized in table 2.

The superpotential is then given exactly as in (4.1). For convenience we rewrite it here:

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\,(\lambda_1 h_+ h_- - \lambda_2 X^2) + \\ + y_1 \sum_{i=1}^{6} n^i_1\, h_+\, \chi^i_- + y_2 \sum_{i=1}^{3} n^i_2\, h_-\, \chi^i_+ \,. \quad (5.3)$$

	SM	U(1)
$\bar{d} \equiv l^{i=1..3}_-$	$(\bar{3}, 1, -1/3)$	-1
$\chi^{i=1..3}_+$	$(3, 1, 1/3)$	-1
$n^{i=1..3}_2$	$(\bar{3}, 1, -1/3)$	0

Table 2: Additional fields: down antiquarks we do not show SM fields that are neutral under the U(1)).

The fields χ^i_-, n^i_1 (χ^i_+, n^i_2) all become heavy, with mass $y_1 h$ ($y_2 h$). The sleptons and down antiquarks obtain the following soft masses

$$\begin{aligned} m^2_{slepton} &= m^2_{gauge} + 9m^2_y|_{y=y_1} - 9m^2_y|_{y=y_2}\,, \\ m^2_{\tilde{d}} &= m^2_{gauge} - 9m^2_y|_{y=y_1} + 9m^2_y|_{y=y_2}\,. \end{aligned} \quad (5.4)$$

Note that because the sleptons and down quarks have opposite U(1) charges, their soft masses obtain opposite contributions from the heavy matter fields.

As explained in the beginning of this section, m^2_{gauge} and m^2_y are negative. Thus, to get a positive contribution to the slepton masses squared, the third term in the first line in (5.4) should overcome the first two. We also point out that m^2_{gauge}, $m^2_{y_1}$, and $m^2_{y_2}$ all start at order $h^2 x^4 = s^4/h^2$. At this order, the dependence on the couplings e, y_i drops out, so that the $\mathcal{O}(x^4)$ terms cancel between $m^2_{y_1}$ and $m^2_{y_2}$.

It is clear from (5.4) that as long as the sleptons obtain a positive contribution to the mass squared, the down-type squarks get a negative contribution. We then have to ensure that the new contribution to the slepton mass is bigger than the one generated directly by anomaly-mediation, whereas the new contribution to the down squark mass is smaller than the one generated by anomaly-mediation. Thus, we need,

$$m^2_{slepton} > \left|m^2_{slepton,AM}\right|\,, \qquad \left|m^2_{\tilde{d}}\right| < m^2_{\tilde{d},AM}\,, \quad (5.5)$$

where $m^2_{slepton,AM}$ and $m^2_{d,AM}$ are the slepton and down squark masses generated by anomaly-mediation in the absence of any heavy thresholds,

$$m^2_{slepton,AM} \sim 10^{-3}\,\frac{F^2}{16\pi^2}\,, \qquad m^2_{d,AM} \sim 10^{-1}\,\frac{F^2}{16\pi^2}\,. \tag{5.6}$$

Thus, the different couplings need to be tuned to satisfy this relation. It turns out that the tuning required is not drastic. First, no large hierarchy is required between the couplings e, y_1 and y_2 to obtain a positive $m^2_{slepton}$. Second, to satisfy (5.5), the coupling λ_0, which controls the size of h, can vary within an overall factor of around five.

We now turn to consider the SM Yukawa couplings. Since both the SU(2)-doublet and -singlet leptons have U(1) charge +1, and the down quarks have U(1) charge -1, the lepton and down Yukawas are not neutral under the U(1). These Yukawas can however arise from non-renormalizable terms,

$$\frac{1}{M}\,h_+\,H_d\,Q\,\bar{d} + \frac{1}{M^2}\,h_-^2\,H_d\,L\,\bar{l}\,, \tag{5.7}$$

where M is some higher scale. We are thus led to a model with nontrivial flavor structure, where the up Yukawas have no suppression, the down Yukawas are suppressed by one power of h/M, and the lepton Yukawas are suppressed by two powers of h/M.

5.2 U(1)-charged leptons and first generations quarks

An alternative to assigning U(1) charge -1 to the SM down-type antiquarks, is to assign charge -1 to the first and second generation doublet quarks. We then have fields l^i_+, χ^i_- and n^i as in the previous subsection, as well as the the first and second generation quarks, which we identify with $l^{i=1,2}_- = (3,2,1/6,-1)$, $\chi^{i=1,2}_+ = (3,2,-1/6,1)$ and $n^{i=1,2}_2 = (3,2,1/6,0)$. We summarize the additional field content of this model in table 3.

	SM	U(1)
$Q \equiv l^{i=1,2}_-$	$(3,2,1/6)$	-1
$\chi^{i=1,2}_+$	$(\bar{3},2,-1/6)$	1
$n^{i=1,2}_2$	$(3,2,1/6)$	0

Table 3: Additional fields: 1st and 2nd generation quarks we do not show SM fields that are neutral under the U(1)).

The sleptons and first and second generation squarks now obtain the following contributions to their masses-squared:

$$\begin{aligned} m^2_{slepton} &= m^2_{gauge} + 9m^2_y|_{y=y_1} - 12m^2_{y_2}|_{y=y_2}\,, \\ m^2_d &= m^2_{gauge} - 9m^2_y|_{y=y_1} + 12m^2_y|_{y=y_2}\,. \end{aligned} \tag{5.8}$$

Again, the third term on the first line gives a positive contribution, with the two other contributions negative. As in the last subsection, we can tune the various couplings so that the total slepton mass is positive, and the squark mass receives only a small (negative) correction.

The lepton Yukawa couplings again arise from non-renormalizable operators, and are suppressed by two powers of h/M. As for the quark Yukawa couplings, the third generation term is not suppressed, whereas the first two generations are suppressed by one power of h/M.

Note that, unlike in the previous subsection, the soft masses are no longer flavor-blind: the soft masses of the first two generation squarks receive negative corrections and are smaller than the soft masses of the third generation squarks. However, these corrections can be chosen to be small, with no severe tuning of parameters. In addition, constraints on FCNC processes which involve the third generation are typically weaker.

6. The μ-term from anomaly mediated SUSY breaking

We now turn to the μ-term problem in AMSB. As has been noted in [1] this problem is much less severe than in gauge mediated models, and indeed several mechanisms generating μ and B terms of the correct order of magnitude have been proposed recently [1, 4, 8]. Here we propose a solution to this problem based on the use of the higher order (in SUSY breaking) correction as well as the observation that in AMSB models it is easy to generate a scale which is somewhat larger than gravitino mass, $m_{3/2} \sim F$.

We introduce the superpotential

$$W = \lambda_H S H_u H_d + \lambda_S S^3 + \lambda_N S N^2 + M N^2 \,, \tag{6.1}$$

where the mass parameter M is assumed to be generated dynamically as in section 4, $M = F/y$ for some coupling constant y. In addition, the scalar components of N have soft tree level contributions to their masses, MF. Generally, the Higgs boson vevs lead to a vev (and therefore an effective μ term) for S through the superpotential (6.1). We will argue shortly that such a contribution does not affect the conclusions we will draw, and therefore, will neglect it throughout our discussion.

Anomaly mediation generates a positive contribution to the S singlet mass squared of the order

$$m^2_{AM} \sim \frac{\lambda_S^4 + \lambda_H^4}{(16\pi^2)^2} F^2 \,. \tag{6.2}$$

Here and throughout this section we omit some order one numerical coefficients. On the other hand, a one loop negative mass squared for the singlet is generated due to the non-decoupling of the heavy states,

$$m^2_{F^4} \sim -\frac{\lambda_N^2}{16\pi^2} \frac{F^4}{M^2} = -\frac{\lambda_N^2}{16\pi^2} y^2 F^2 \,. \tag{6.3}$$

It is easy to see that the singlet mass will be negative as long as[6]

$$\lambda_N^2 y^2 > \frac{\lambda_H^4 + \lambda_S^4}{16\pi^2}\,, \tag{6.4}$$

where the right hand side indicates the order of magnitude only.

As a result both the scalar and the auxiliary components of S acquire vevs

$$\begin{aligned} S &\sim \frac{1}{4\pi}\frac{\lambda_N y}{\lambda_S} F\,,\\ F_S &\sim \frac{1}{16\pi^2}\frac{\lambda_N^2 y^2}{\lambda_S} F^2\,. \end{aligned} \tag{6.5}$$

Substituting these vevs into the superpotential (6.1) we find that both μ and B are generated

$$\begin{aligned} \mu &\sim \frac{1}{4\pi}\frac{\lambda_H}{\lambda_S} y\lambda_N F\,,\\ B &\sim \frac{1}{16\pi^2}\frac{\lambda_H}{\lambda_S} y^2\lambda_N^2 F^2\,. \end{aligned} \tag{6.6}$$

After S acquires a vev there is an additional contribution to B arising from the $S\,H_u\,H_d$ A-term, however, it is negligible when (6.4) is satisfied. It is easy to see that B and μ^2 are of the same order if

$$\frac{\lambda_H}{\lambda_S} \sim \mathcal{O}(1)\,. \tag{6.7}$$

We further need to require that the μ term is of the order of the weak scale,

$$\frac{1}{4\pi}\frac{\lambda_H}{\lambda_S} y\lambda_N F \sim \frac{\alpha_2}{4\pi} F\,. \tag{6.8}$$

This requirement together with (6.7) gives a condition on two Yukawa couplings $\lambda_N y \sim \alpha_2$. We note that this condition is quite compatible with the requirement that S has a negative mass squared.

Finally we observe that the Higgs vevs generate a mass term for S. This mass contribution is below the negative mass (6.3) by roughly $\lambda_H\, H_U/\mu$. Since λ_H can be arbitrary as long as it is comparable with λ_S, such a contribution is small compared to the negative mass generated by non-decoupling effects (which we can arrange to be between electroweak scale and 1TeV). Even in the case $\lambda_H \sim 1$, our qualitative conclusions remain valid, and both a μ and a B term of the correct order of magnitude are generated.

Having established that eq. (6.6) gives a leading contribution to μ and B it is possible to show that the physical phase $\phi = \arg(B\mu^* M_\lambda^*)$ vanishes. Here M_λ is the gaugino mass. Thus, this sector of the theory does not lead to a SUSY CP problem.

[6] Remember that N is heavy, and as a result λ_N does not contribute to the S mass at order F^2.

To conclude our discussion of the μ term, we observe that the superpotential (6.1) could be introduced in a gauge mediated model, with N being a messenger field. However, in calculable models of gauge mediation the scale of supersymmetry breaking is relatively large, while the messenger mass is at most suppressed by several loop factors relative to this scale. As a result the higher order contributions used here are too small to generate electroweak scale parameters.[7] In principle it is possible to generate a small mass for the messengers, however, this requires the introduction of a quite complicated structure and explicit mass scales, unlike with anomaly mediation where a mass scale somewhat larger than the SUSY breaking scale of the visible sector can naturally be generated.

7. Conclusions

In this paper, we studied the decoupling of heavy thresholds in theories with anomaly-mediated supersymmetry breaking. To leading order in the supersymmetry breaking, such thresholds decouple. That is, the anomaly-mediated supersymmetry breaking terms at some low scale are independent of whether or not there are supersymmetric thresholds above that scale. These soft terms are thus quite robust.

It is possible to see this decoupling in several ways. For example, it can be understood as a cancellation between the following two quantities: The first is the contribution of the heavy fields to the anomaly mediated soft terms in the full theory. Recall that the AM soft terms depend on the beta function of the theory, which in the full theory reflects the presence of the heavy fields. The second is the direct radiative contribution, through gauge or Yukawa interactions, of the heavy fields to the soft terms of the light fields. This contribution is generated when the heavy fields are integrated out. These two contributions cancel exactly, so that below the scale of the heavy fields, they leave no trace on the soft terms at leading order in SUSY breaking. That these two contributions exactly cancel can be seen on a case by case basis, but it is most simply seen from the fact that the direct gauge- or Yukawa-mediated contributions of the heavy fields can be read off the wave function renormalizations [5, 6] in precisely the same way as the AM contributions.

Alternatively, the decoupling can be seen as a cancellation between "real" fields and their regulators.

When do heavy threshold not decouple? One obvious possibility is that they are not truly heavy [4]. That is, there is some light modulus associated with the heavy threshold whose mass comes mainly from SUSY breaking effects. It is worth

[7] This is not a problem in strongly coupled gauge mediated models with a low SUSY breaking scale. However, such models are non-calculable, and it is not possible to quantitatively analyze their spectrum.

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pointing out that approximate decoupling persists even for a modulus much lighter than the SUSY breaking scale F so long as its mass is primarily determined by supersymmetric parameters.

However, there is additional non-decoupling even when all the heavy fields are truly heavy. That is because the purely anomaly-mediated soft terms only appear at leading order in the SUSY breaking. For example, scalar masses squared are order F^2. In contrast, as we integrate out some heavy fields, they give direct contributions, again through loops, to the soft terms to all orders in the SUSY breaking. Scalar masses squared now have contributions of order F^4/M^2, where M is the heavy threshold. Moreover, these contributions typically appear at lower order in the loop expansion. For scalar masses squared, they can appear at one-loop, whereas the AM soft masses are two-loop contributions.

Having established that heavy supersymmetric thresholds do affect the soft terms at order F^4, we then use this fact to generate positive slepton masses. If the only source of SUSY breaking in the SM is anomaly mediation, and if there are no supersymmetric thresholds, the slepton masses squared are negative. However, we can charge the leptons (and another subset of SM fields) under a new U(1) gauge symmetry, which is broken at a scale somewhat above the visible sector SUSY-breaking scale, and add some fields that obtain supersymmetric masses. In the presence of anomaly-mediation, these heavy fields also acquire SUSY-breaking masses, and contribute to slepton masses at order F^4. Interestingly, we are led to a model with some non-trivial flavor structure. Unfortunately, this model is not consistent with grand unification.

As another model building application, we used the F^4 contributions of heavy supersymmetric thresholds to generate acceptable μ- and B-terms. As we saw, this can be done quite simply in models of anomaly mediation, unlike in the case of gauge-mediation. In the latter case, an acceptable μ term typically leads to a B term that is too large.

In both these model building examples, we also use a simple mechanism that dynamically generates a scale that is naturally somewhat above the SUSY breaking scale through anomaly-mediation. We expect this fact to be useful for further model building applications.

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